Fast Anisotropic Metasurface Analysis in FDTD using Surface Susceptibility Model

Xiao Jia, Fan Yang*, Maokun Li, Shenheng Xu

Department of Electronic Engineering, Tsinghua University, Beijing, China

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Abstract

The FDTD methods combined with Surface Susceptibility Model (FDTD-SSM), compared to traditional brute force metasurface simulation, have been proved efficient and accurate for fast metasurface analysis. However, the present work is limited to scattering elements that are either isotropic or that have diagonal surface susceptibilities. This paper proposes a necessary extensions of Finite Difference Time Domain equations to analyze general anisotropic metasurfaces which have off-diagonal surface susceptibility tensors. At last, the Surface Susceptibility Model simulation results are validated by geometrical structure simulation results.

Keyword: Metasurfaces, GSTCs, FDTD, surface susceptibilities

Biography

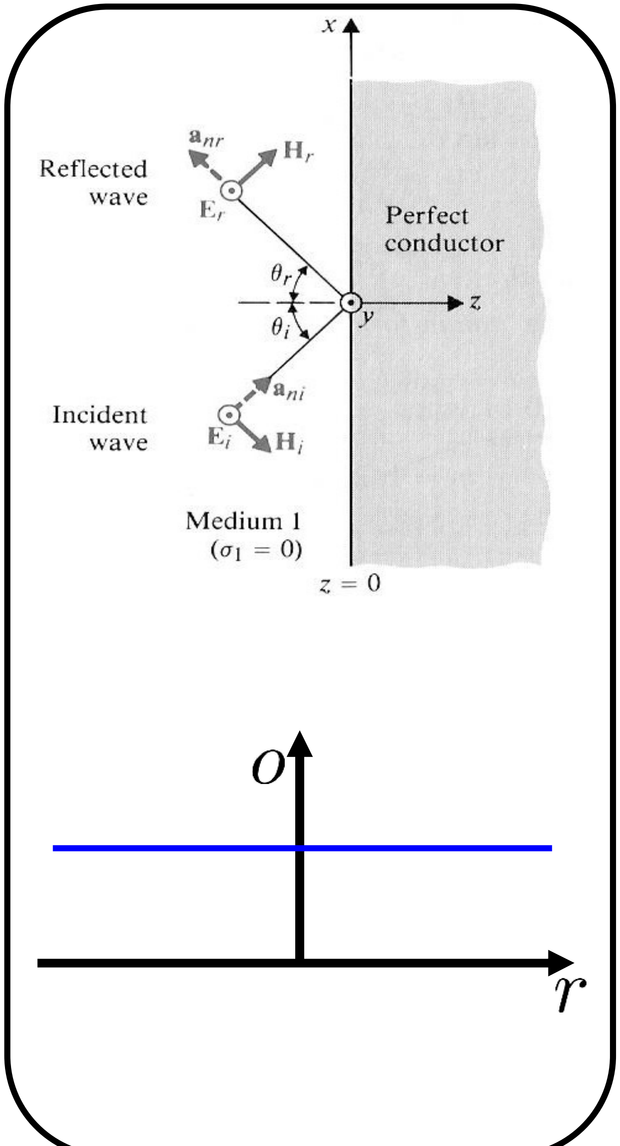
Xiao Jia received the B.S. degree from Northwestern Polytechnical University, Xi'an, China, the M.S. degree form University of Chinese Academy of Sciences, Beijing, China, in 2013 and 2016, respectively, She received the Ph.D. degree from Tsinghua University, Beijing, China, in 2020, under the supervision of Prof. Fan Yang. From 2018 to 2019, she was a visiting student at Polytechnique Montréal, Montréal, Québec, Canada, with the supervision of Prof. Christophe Caloz.

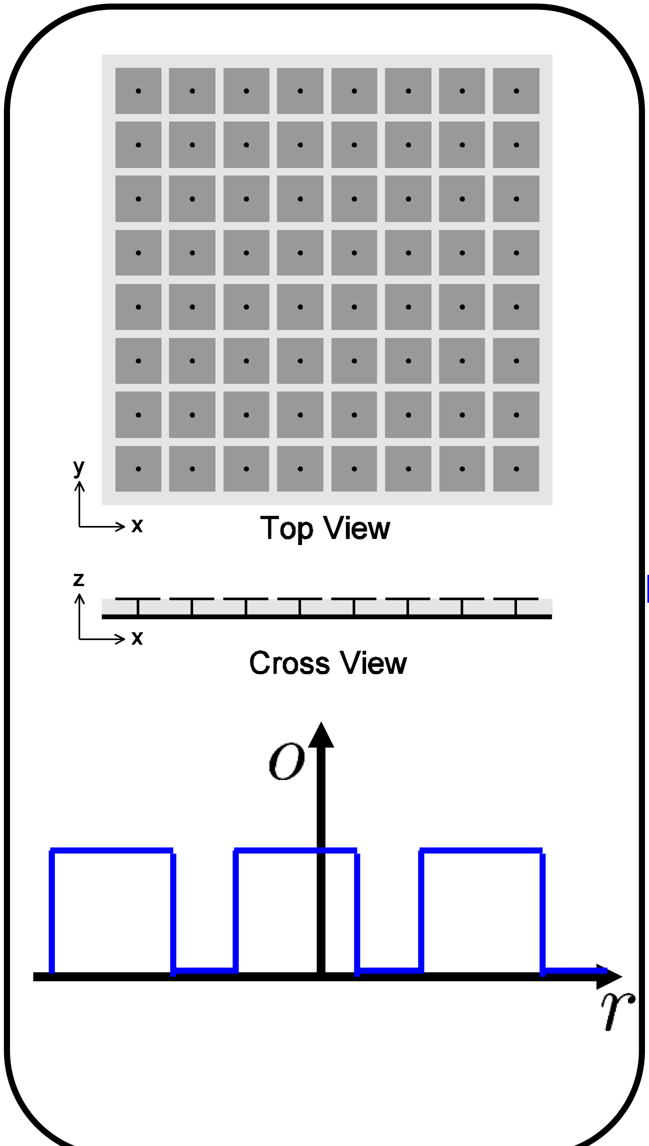
She is currently a lecturer at Beijing Jiaotong University, Beijing, China. Her current research interests include computational electromagnetics, metasurfaces, computational electromagnetics, reflectarray and transmitarray.

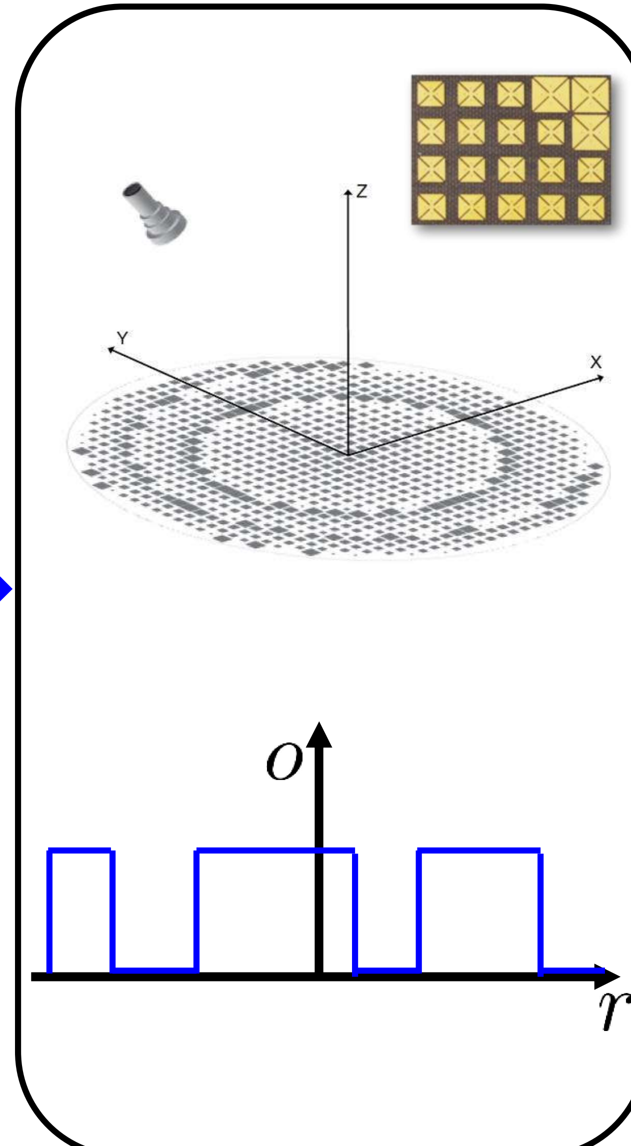
Outline

- Introduction of Metasurfaces
- Modeling of metasurfaces
- FDTD-SSM Algorithm
- Numerical Experiments
- Conclusion

Development of Metasurfaces



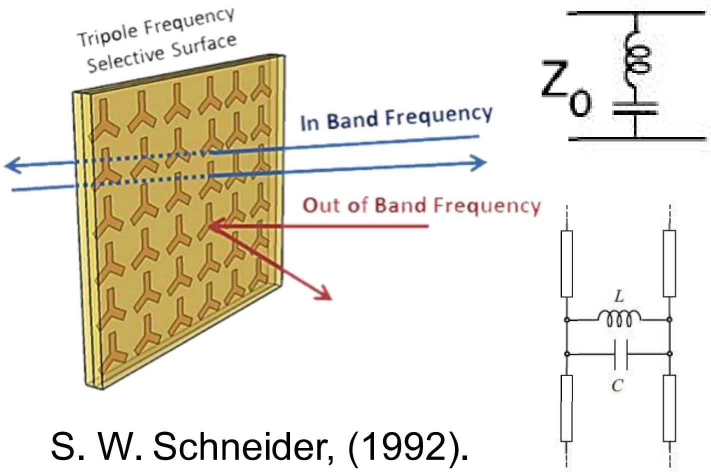




Ref: Yang F, Rahmat-Samii Y. Reflection phase characterizations of the EBG ground plane for low profile wire antenna applications[J]. IEEE Transactions on antennas and propagation, 2003, 51(10): 2691-2703.
Ref: Nayeri P, Yang F, Elsherbeni A Z. Reflectarray antennas: theory, designs, and applications[M]. John Wiley & Sons, 2018.

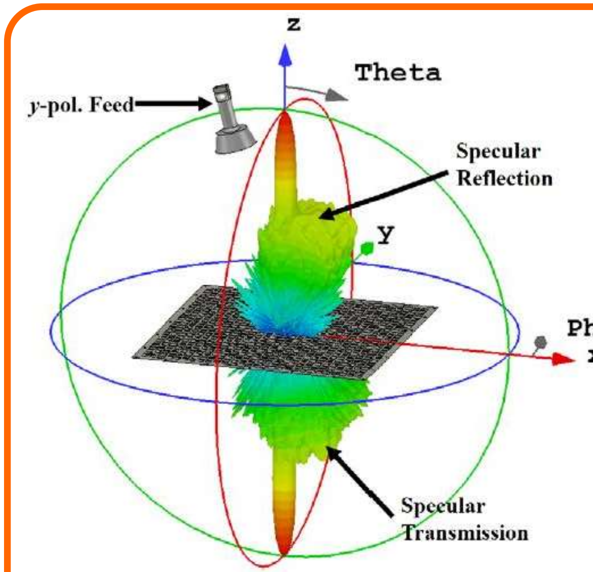
Applications of Metasurfaces

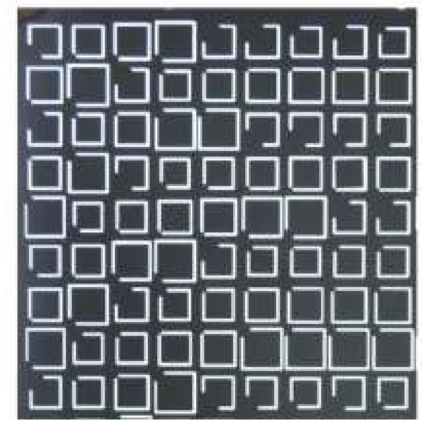
Frequency Selective Surfaces (FSSs)



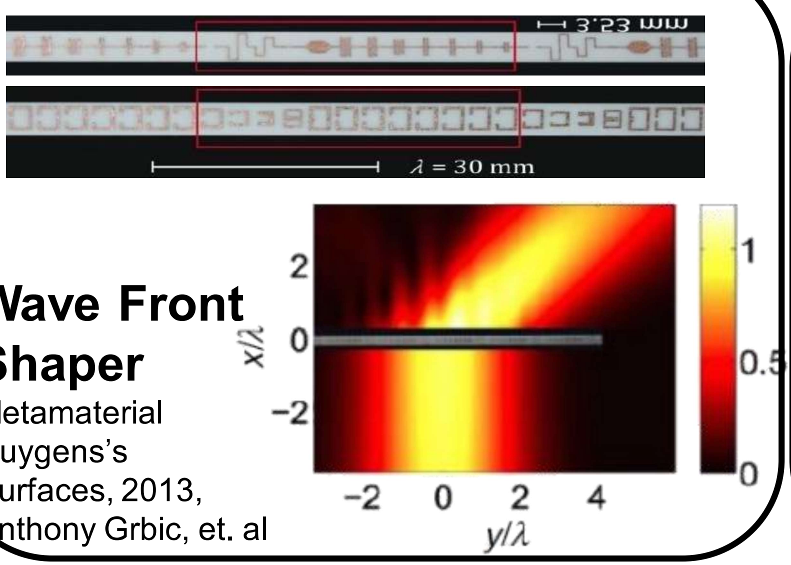
S. W. Schneider, (1992).

High-Gain Transmit-Reflect-Array



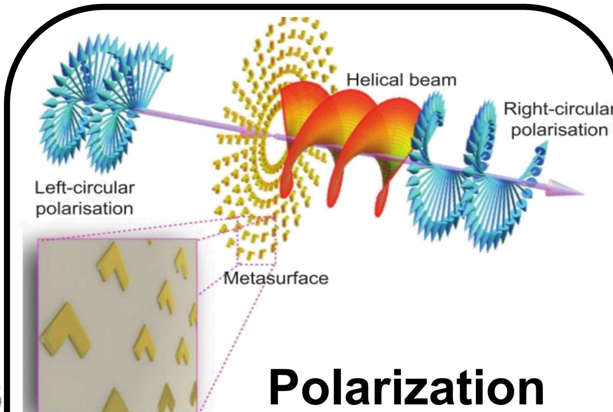


Fan Yang *et al (2018).*



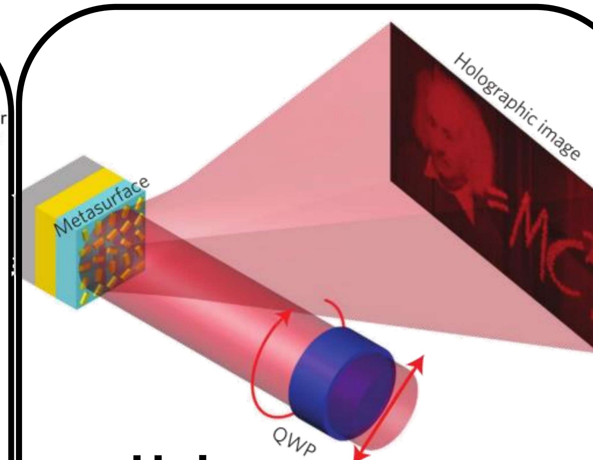
Wave Front Shaper

Metamaterial Huygens's Surfaces, 2013, Anthony Grbic, et. al



Polarization Controller

C. Pfeiffer *et al.*, PRA, (2014).

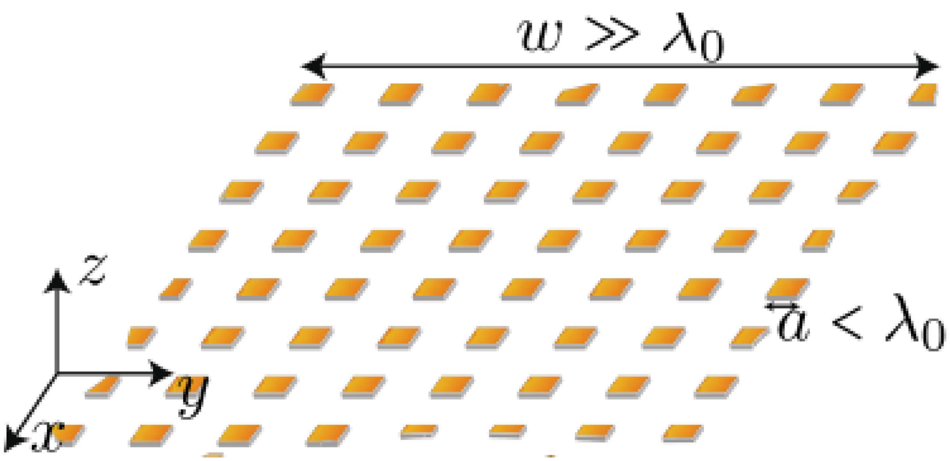


Hologram

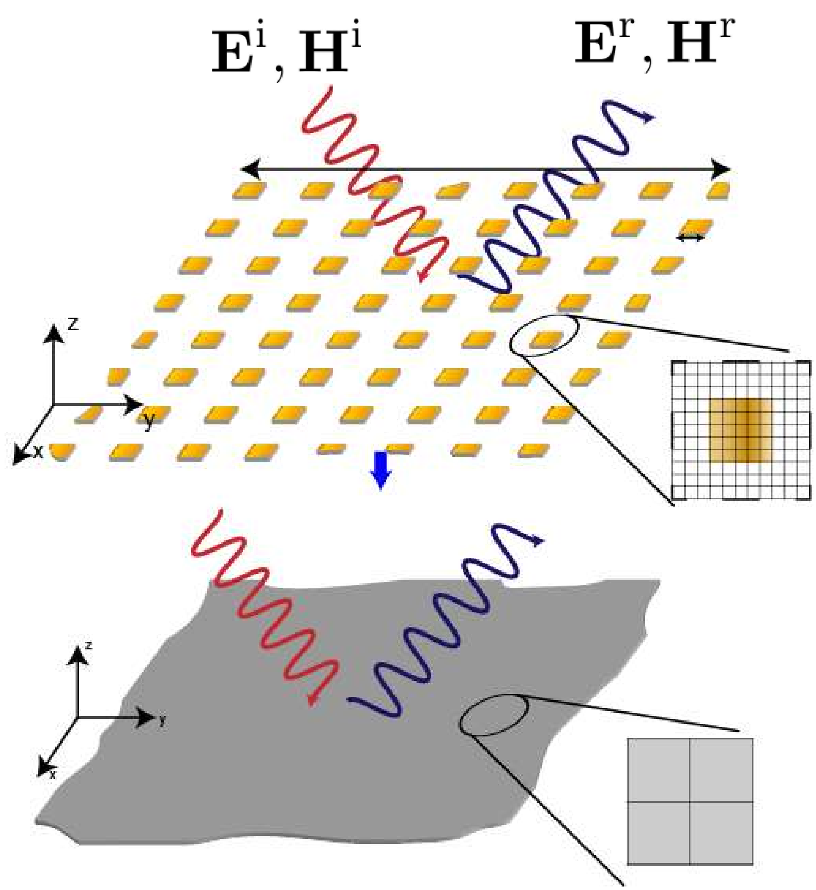
G. Zheng *et al.*, Nat. Nano, (2015)

Computational Challenges and Our Solutions

Electrically large global size



Subwavelength elements



Remarks:

- Extremely dense meshes and huge number of unknowns leads to enormous computational problems, using brute force simulations.
- We propose to replace physical scattering-particle unit cells of metasurfaces by their corresponding homogenized surface model.

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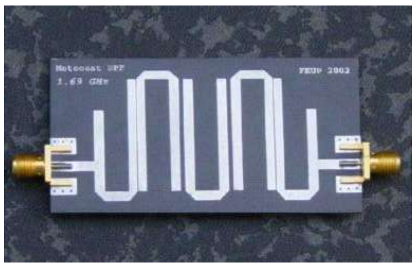
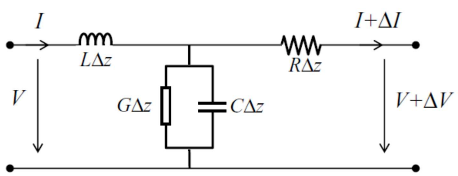
System level models: from 1D to 3D

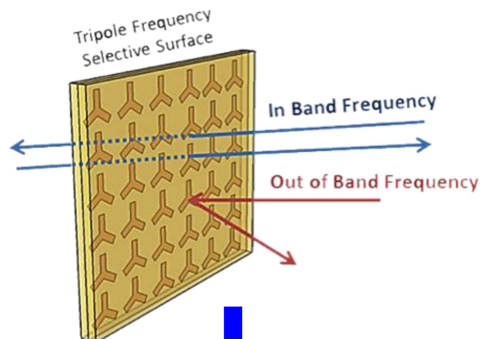
Engineers are constantly challenged with the temptation to search for optimum solutions for *complex engineering system designs*. Establishing *system level models* is a popular and efficient way.

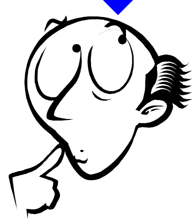
1D
Transmission Line



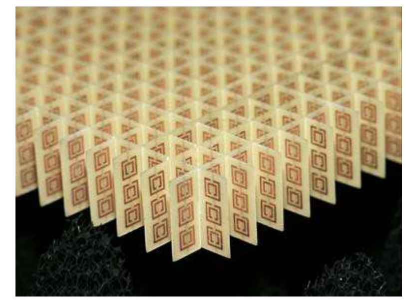


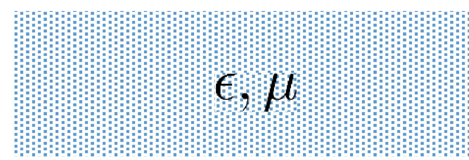
2D
Metasurface





3D
Metamaterial

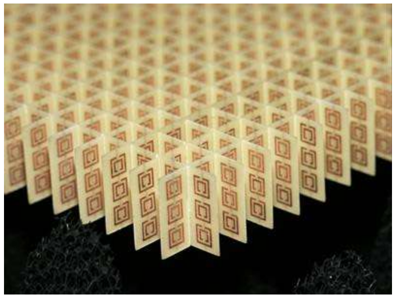


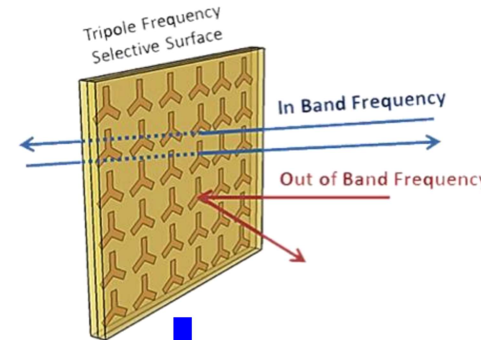


Remarks:

- The input and output of 1D case is scalar variable $V(t), I(t)$, while ours is vectors $\vec{E}(\vec{r},t)$, $\vec{H}(\vec{r},t)$.
- Apart from magnitude, phase and frequency, we additionally considered polarization and propagation direction.

From 3D Metamaterial to 2D Metasurface





ϵ, μ

ξ, ζ

characterization

$\chi_{\rm ee}, \chi_{\rm em}$

$\chi_{\rm mm}, \chi_{\rm me}$

$$\mathbf{D} = \bar{\bar{\epsilon}} \cdot \mathbf{E} + \bar{\bar{\xi}} \cdot \mathbf{H}$$
$$\mathbf{B} = \bar{\bar{\mu}} \cdot \mathbf{H} + \bar{\bar{\zeta}} \cdot \mathbf{E}$$

relationship

$$\mathbf{P}_{\rm s} = \epsilon_0 \bar{\bar{\chi}}_{\rm ee} \mathbf{E}_{\rm av} + \sqrt{\mu_0 \epsilon_0} \bar{\bar{\chi}}_{\rm em} \mathbf{H}_{\rm av}$$
$$\mathbf{M}_{\rm s} = \sqrt{\epsilon_0 / \mu_0} \bar{\bar{\chi}}_{\rm me} \mathbf{E}_{\rm av} + \bar{\bar{\chi}}_{\rm mm} \mathbf{H}_{\rm av}$$

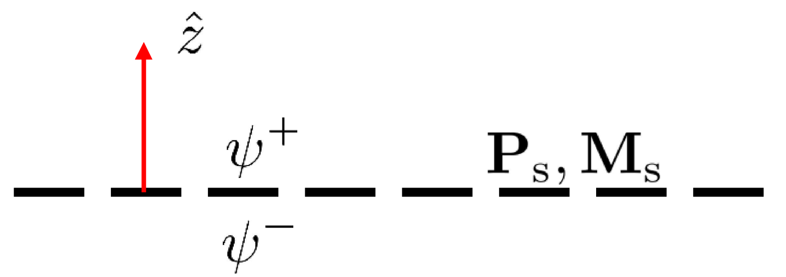
$\mathbf{E}, \mathbf{H}$

Analysis

$\mathbf{E}, \mathbf{H}$

2D Form of Surface Susceptibility Tensor

$$\mathbf{P}_s = \epsilon_0 \overline{\overline{\chi}}_{ee} \mathbf{E}_{av} + \sqrt{\mu_0 \epsilon_0}\, \overline{\overline{\chi}}_{em} \mathbf{H}_{av}$$
$$\mathbf{M}_s = \sqrt{\epsilon_0/\mu_0}\, \overline{\overline{\chi}}_{me} \mathbf{E}_{av} + \overline{\overline{\chi}}_{mm} \mathbf{H}_{av}$$



$$\mathbf{\Psi}_{av} = \frac{\Psi^+ + \Psi^-}{2}, \quad \mathbf{\Psi} = \mathbf{E}, \mathbf{H},$$

- Assumption 1: infinitely thin metafilm: $\rightarrow P_z = 0, M_x = 0, M_y = 0$
- Assumption 2: reciprocity $\rightarrow \overline{\overline{\chi}}_{ee}^T = \overline{\overline{\chi}}_{ee}, \quad \overline{\overline{\chi}}_{mm}^T = \overline{\overline{\chi}}_{mm}, \quad \overline{\overline{\chi}}_{me}^T = -\overline{\overline{\chi}}_{em}$
- Assumption 3: magnetoelectric terms are zero: $\rightarrow \overline{\overline{\chi}}_{me} = 0, \overline{\overline{\chi}}_{em} = 0$

$$\overline{\overline{\chi}}_{ee} = \begin{bmatrix} \chi_{ee}^{xx} & \chi_{ee}^{xy} & \chi_{ee}^{xz} \\ \chi_{ee}^{yx} & \chi_{ee}^{yy} & \chi_{ee}^{yz} \\ \chi_{ee}^{zx} & \chi_{ee}^{zy} & \chi_{ee}^{zz} \end{bmatrix} \quad \overline{\overline{\chi}}_{mm} = \begin{bmatrix} \chi_{mm}^{xx} & \chi_{mm}^{xy} & \chi_{mm}^{xz} \\ \chi_{mm}^{yx} & \chi_{mm}^{yy} & \chi_{mm}^{yz} \\ \chi_{mm}^{zx} & \chi_{mm}^{zy} & \chi_{mm}^{zz} \end{bmatrix}$$

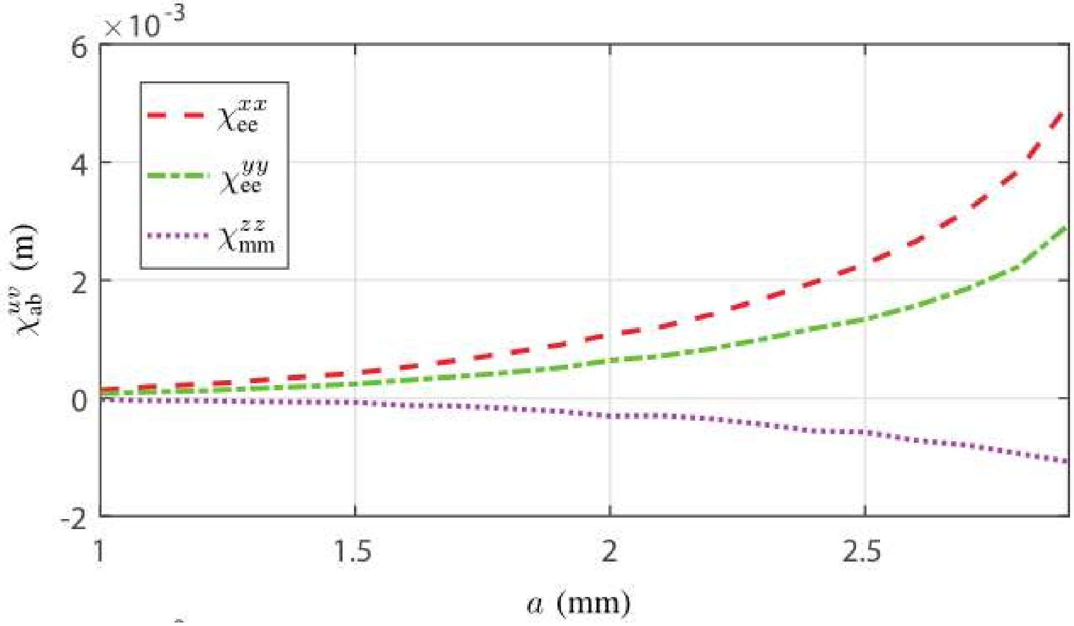
$$\overline{\overline{\chi}}_{em} = \begin{bmatrix} \chi_{em}^{xx} & \chi_{em}^{xy} & \chi_{em}^{xz} \\ \chi_{em}^{yx} & \chi_{em}^{yy} & \chi_{em}^{yz} \\ \chi_{em}^{zx} & \chi_{em}^{zy} & \chi_{em}^{zz} \end{bmatrix} \quad \overline{\overline{\chi}}_{me} = \begin{bmatrix} \chi_{me}^{xx} & \chi_{me}^{xy} & \chi_{me}^{xz} \\ \chi_{me}^{yx} & \chi_{me}^{yy} & \chi_{me}^{yz} \\ \chi_{me}^{zx} & \chi_{me}^{zy} & \chi_{me}^{zz} \end{bmatrix}$$

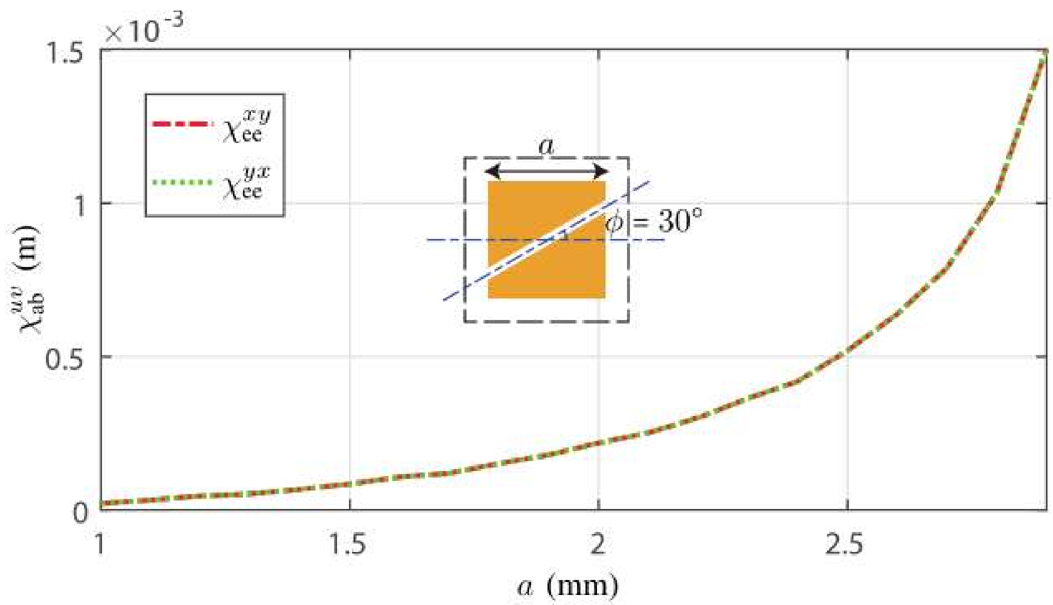
$\Rightarrow$

$$\overline{\overline{\chi}}_{ee} = \begin{bmatrix} \chi_{ee}^{xx} & \chi_{ee}^{xy} & 0 \\ \chi_{ee}^{yx} & \chi_{ee}^{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\overline{\overline{\chi}}_{mm} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \chi_{mm}^{zz} \end{bmatrix}$$

Anisotropic SSM extraction





$$\overline{\overline{\chi}}_{ee} = \begin{bmatrix} \chi^{xx}_{ee} & \chi^{xy}_{ee} & 0 \\ \chi^{yx}_{ee} & \chi^{yy}_{ee} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \overline{\overline{\chi}}_{mm} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \chi^{zz}_{mm} \end{bmatrix}$$

TE $\theta = 0$ TM $\theta = 0$

$$\chi^{xx}_{ee} = \frac{2(T_c^{TE} - T_c^{TE}T_c^{TM} + T_X^{TE}T_X^{TM})}{j\omega\epsilon_0\eta(T_c^{TE}T_c^{TM} - T_X^{TE}T_X^{TM})}|_{\theta=0},$$

$$\chi^{yy}_{ee} = \frac{2(T_c^{TM} - T_c^{TE}T_c^{TM} + T_X^{TE}T_X^{TM})}{j\omega\epsilon_0\eta(T_c^{TE}T_c^{TM} - T_X^{TE}T_X^{TM})}|_{\theta=0},$$

$$\chi^{xy}_{ee} = \frac{-2T_X^{TE}}{j\omega\epsilon_0\eta(T_c^{TE}T_c^{TM} - T_X^{TE}T_X^{TM})}|_{\theta=0},$$

$$\chi^{yx}_{ee} = \frac{-2T_X^{TM}}{j\omega\epsilon_0\eta(T_c^{TE}T_c^{TM} - T_X^{TE}T_X^{TM})}|_{\theta=0}$$

TE $\theta = \theta_0$

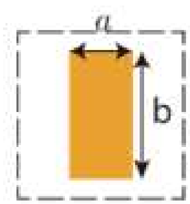
$$\chi^{zz}_{mm} = \frac{(2 - 2T_c^{TE})\cos(\theta) - j\omega\epsilon_0\eta(\chi^{yx}_{ee}T_X^{TE}\cos(\theta) + \chi^{yy}_{ee}T_c^{TE})}{jk_xT_c^{TE}\sin(\theta)}|_{\theta=\theta_0}$$

Bi-axial Susceptibility Tensor

$$\overline{\overline{\chi}}_{\mathrm{ee}} = \begin{bmatrix} \chi_{\mathrm{ee}}^{xx} & \chi_{\mathrm{ee}}^{xy} & 0 \\ \chi_{\mathrm{ee}}^{yx} & \chi_{\mathrm{ee}}^{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\overline{\overline{\chi}}_{\mathrm{mm}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \chi_{\mathrm{mm}}^{zz} \end{bmatrix}$$

Bi-axial



$$\overline{\overline{\chi}}_{\mathrm{ee}} = \begin{bmatrix} \chi_{\mathrm{ee}}^{xx} & 0 & 0 \\ 0 & \chi_{\mathrm{ee}}^{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\overline{\overline{\chi}}_{\mathrm{mm}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \chi_{\mathrm{mm}}^{zz} \end{bmatrix}$$

Remarks:

1. The unknowns are further reduced to 3 for bi-axial case.
2. Bi-axial case means that the structure do not generate cross polarization and transformation between electric to magnetic fields.

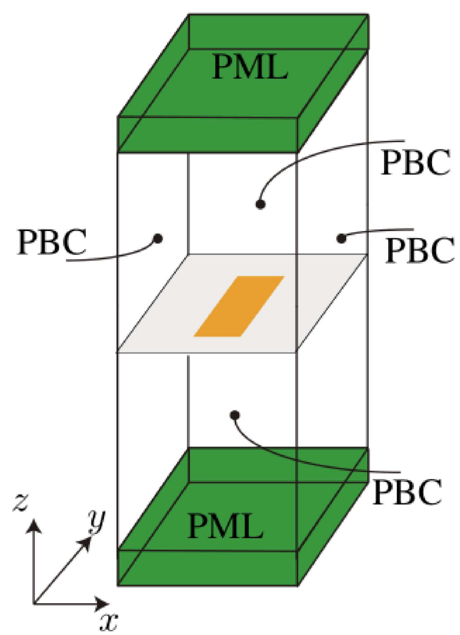
Surface susceptibility for bi-axial case

The relationship between surface susceptibility and reflection coefficients:

$$\mathrm{TM}_z : R^{\theta}_{\mathrm{TM}} = \frac{j\omega\epsilon_0\chi^{xx}_{\mathrm{ee}}\eta\cos\theta_{\mathrm{i}}}{-2 - j\omega\epsilon_0\chi^{xx}_{\mathrm{ee}}\eta\cos\theta_{\mathrm{i}}}$$

$$\mathrm{TE}_z : R^{\theta}_{\mathrm{TE}} = \frac{j\omega\epsilon_0\chi^{yy}_{\mathrm{ee}}\eta + jk_0\sin^2\theta_{\mathrm{i}}\chi^{zz}_{\mathrm{mm}}}{-2\cos\theta_{\mathrm{i}} - j\omega\epsilon_0\chi^{yy}_{\mathrm{ee}}\eta - jk_0\sin^2\theta_{\mathrm{i}}\chi^{zz}_{\mathrm{mm}}}$$

Three sets of simulation is required to extract the surface susceptibilities for bi-axial case:

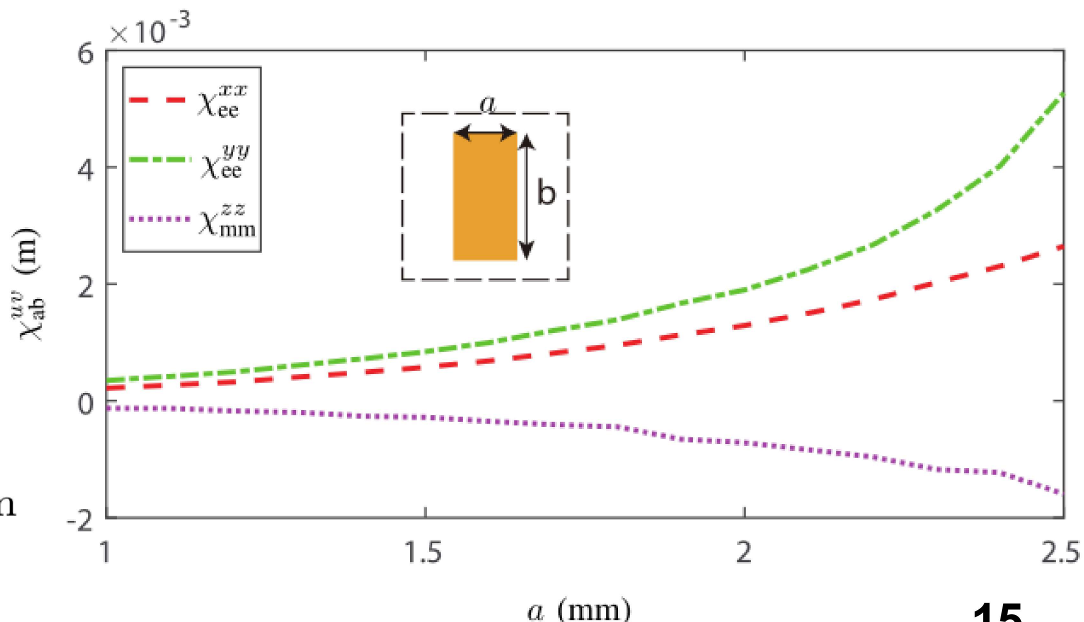


$$\overline{\overline{\chi}} = f(R)$$

$$\mathrm{TM} : \theta_{\mathrm{i}} = 0 \rightarrow \chi^{xx}_{\mathrm{ee}}$$

$$\mathrm{TE} : \theta_{\mathrm{i}} = 0 \rightarrow \chi^{yy}_{\mathrm{ee}}$$

$$\mathrm{TE} : \theta_{\mathrm{i}} = \theta_0 \rightarrow \chi^{zz}_{\mathrm{mm}}$$



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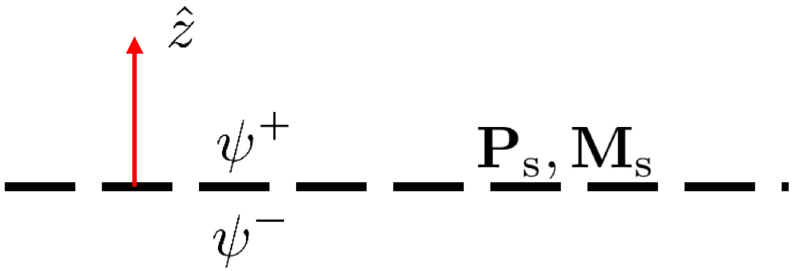
SSM & Surface Currents



$$\mathbf{\Psi}_{av} = \frac{\Psi^+ + \Psi^-}{2}, \quad \mathbf{\Psi} = \mathbf{E}, \mathbf{H},$$

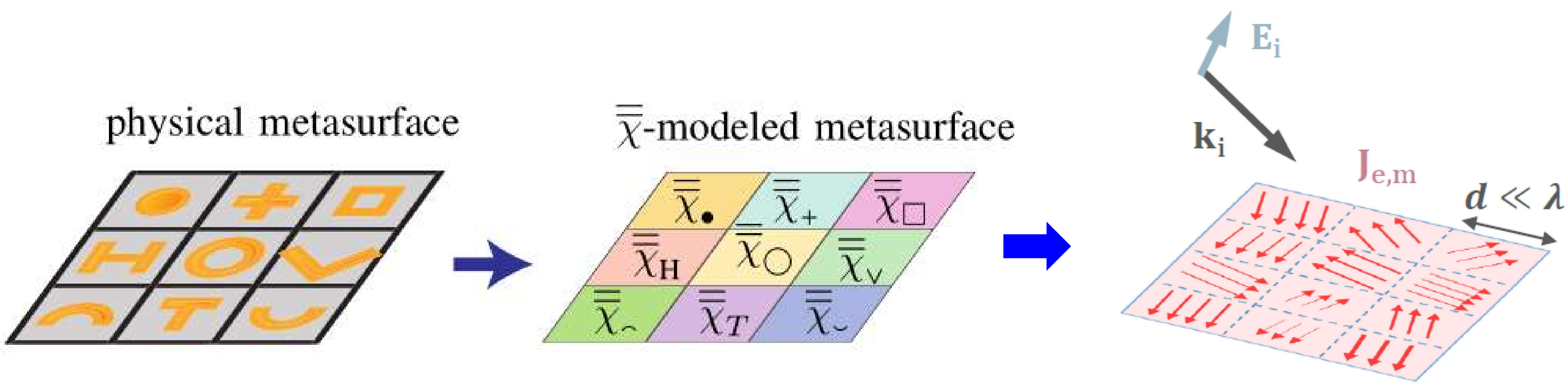
$$\mathbf{P}_s = \epsilon_0 \overline{\overline{\chi}}_{ee} \mathbf{E}_{av} + \sqrt{\mu_0 \epsilon_0} \overline{\overline{\chi}}_{em} \mathbf{H}_{av}$$

$$\mathbf{M}_s = \sqrt{\epsilon_0 / \mu_0} \overline{\overline{\chi}}_{me} \mathbf{E}_{av} + \overline{\overline{\chi}}_{mm} \mathbf{H}_{av}$$

Surface currents:

$$\mathbf{J}_{s,tot} = \mathbf{J}_{s,f} + \mathbf{J}_{s,p} + \mathbf{J}_{s,m} = \mathbf{J}_{s,f} + \frac{\partial \mathbf{P}_{s,\parallel}}{\partial t} + (\nabla \times \mathbf{M}_s)_\parallel \quad (A/m)$$

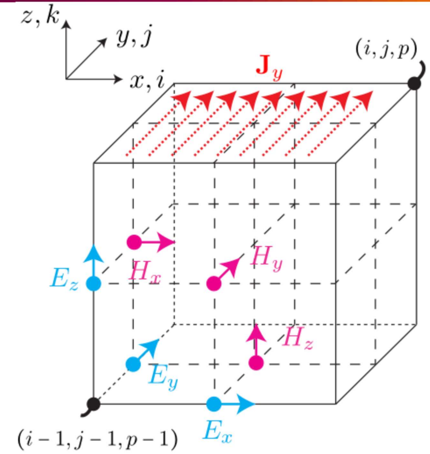
$$\mathbf{K}_{s,tot} = \mathbf{K}_{s,f} + \mathbf{K}_{s,m} + \mathbf{K}_{s,p} = \mathbf{K}_{s,f} + \mu_0 \frac{\partial \mathbf{M}_{s,\parallel}}{\partial t} + [\nabla \times (\mathbf{P}_s / \epsilon_0)]_\parallel \quad (V/m)$$



Ref 1: Idemen, M. Mithat. Discontinuities in the electromagnetic field. Vol. 40. John Wiley & Sons, 2011.
Ref 2: Achouri, Karim, Mohamed A. Salem, and Christophe Caloz. "General metasurface synthesis based on susceptibility tensors." IEEE Transactions on Antennas and Propagation 63.7 (2015): 2977-2991.
Ref 3: Kuester, Edward F., et al. "Averaged transition conditions for electromagnetic fields at a metafilm." IEEE Transactions on Antennas and Propagation 51.10 (2003): 2641-2651.

Surface currents in FDTD

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = \epsilon_0 \iint \frac{\partial \boldsymbol{E}}{\partial t} \cdot d\boldsymbol{s} + \iint \boldsymbol{J}_{\mathrm{s}} \cdot \delta(z) d\boldsymbol{s}$$



Discretization in Yee cell

$$(Hx^{n-1/2}(i,j,p) - Hx^{n-1/2}(i,j,p-1))\triangle x + (Hz^{n-1/2}(i,j,p) - Hz^{n-1/2}(i-1,j,p))\triangle z$$
$$= \epsilon_0 \frac{E_y^n(i,j,p) - E_y^{n-1}(i,j,p)}{\triangle t} \triangle x \triangle z + J_y^{n-1/2}(i,j,p)\triangle x$$

Rearranging

$$E_y^n(i,j,p) = E_y^{n-1}(i,j,p) + \frac{\triangle t}{\epsilon_0}$$
$$\left[\frac{Hx^{n-1/2}(i,j,p) - Hx^{n-1/2}(i,j,p-1)}{\triangle z} - \frac{Hz^{n-1/2}(i,j,p) - Hz^{n-1/2}(i-1,j,p)}{\triangle x}\right]$$
$$- \frac{\triangle t}{\epsilon_0} \frac{J_y^{n-1/2}(i,j,p)}{\triangle z}.$$

Bi-axial SSM in FDTD

$$\mathbf{P} = \epsilon_0 \begin{bmatrix} \chi_{ee}^{xx} & 0 & 0 \\ 0 & \chi_{ee}^{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{E}, \quad \mathbf{M} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \chi_{mm}^{zz} \end{bmatrix} \mathbf{H}$$

$$\boldsymbol{J}_s = \frac{\partial \boldsymbol{P}}{\partial t} + \nabla \times \boldsymbol{M}$$

$$J_y = (\epsilon_0 \chi_{ee}^{yy} \frac{\partial E_y}{\partial t} - \frac{\partial(\chi_{mm}^{zz} H_z)}{\partial x})\hat{y}$$

$$E_y^n(i,j,p) = E_y^{n-1}(i,j,p) + \frac{\triangle t}{\epsilon_0} \left[\frac{Hx^{n-1/2}(i,j,p) - Hx^{n-1/2}(i,j,p-1)}{\triangle z} - \frac{Hz^{n-1/2}(i,j,p) - Hz^{n-1/2}(i-1,j,p)}{\triangle x} \right] - \frac{\triangle t}{\epsilon_0} \frac{J_y^{n-1/2}(i,j,p)}{\triangle z}$$

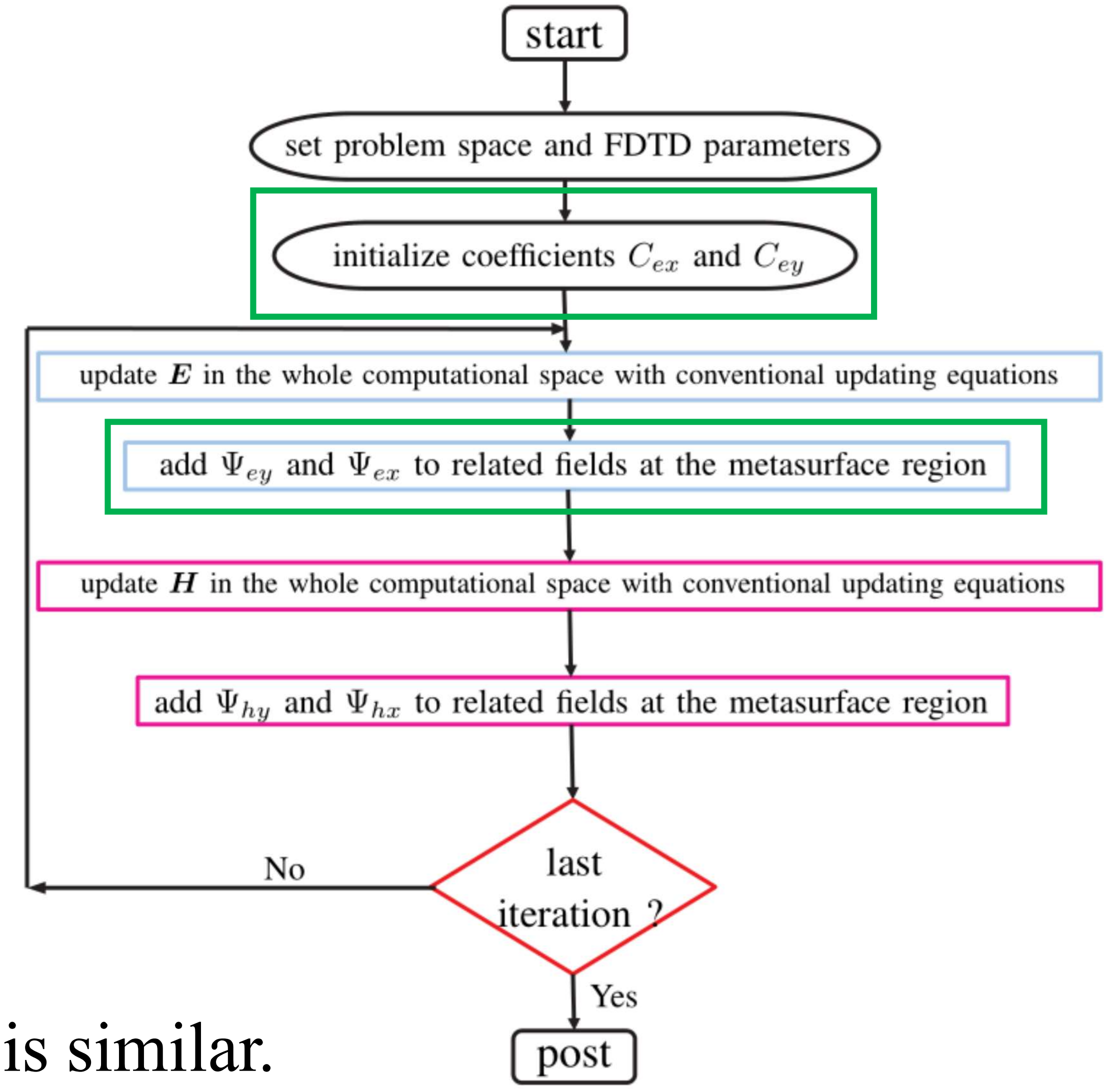
$$E_y^n(i,j,p) = E_y^{n-1}(i,j,p) + \frac{\triangle t \triangle z}{\epsilon_0(\triangle z + \tilde{\chi}_{ee}^{yy}(i,j,p))} \left[\frac{H_x^{n-1/2}(i,j,p) - H_x^{n-1/2}(i,j,p-1)}{\triangle z} - \frac{H_z^{n-1/2}(i,j,p) - H_z^{n-1/2}(i-1,j,p)}{\triangle x} \right] + \frac{\triangle t}{\epsilon_0(\triangle z + \tilde{\chi}_{ee}^{yy}(i,j,p))} \frac{\chi_{mm}^{zz}(i,j,p) H_z^{n-1/2}(i,j,p) - \chi_{mm}^{zz}(i-1,j,p) H_z^{n-1/2}(i-1,j,p)}{\triangle x}$$

Flowchart of FDTD-SSM

$$C_{ey} = \frac{\triangle t \triangle z}{\epsilon_0(\triangle z + \tilde{\chi}_{\mathrm{ee}}^{yy}(i,j,p))}$$

$$\Psi_{ey} = \frac{\triangle t}{\epsilon_0(\triangle z + \tilde{\chi}_{\mathrm{ee}}^{yy}(i,j,p))} \frac{\chi_{\mathrm{mm}}^{zz}(i,j,p) H_z^{n-1/2}(i,j,p) - \chi_{\mathrm{mm}}^{zz}(i-1,j,p) H_z^{n-1/2}(i-1,j,p)}{\triangle x}$$

$$E_y^n(i,j,p) = E_y^{n-1}(i,j,p) + C_{ey} \left[\frac{H_x^{n-1/2}(i,j,p) - H_x^{n-1/2}(i,j,p-1)}{\triangle z} - \frac{H_z^{n-1/2}(i,j,p) - H_z^{n-1/2}(i-1,j,p)}{\triangle x} \right] + \Psi_{ey}$$



The updating procedure of E_x is similar.

Anisotropic FDTD-SSM

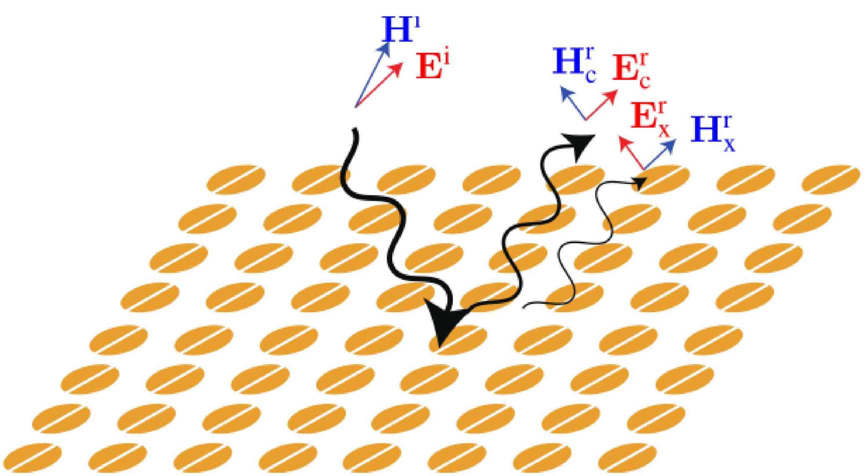
$$\overline{\overline{\chi}}_{\mathrm{ee}} = \begin{bmatrix} \chi_{\mathrm{ee}}^{xx} & \chi_{\mathrm{ee}}^{xy} & 0 \\ \chi_{\mathrm{ee}}^{yx} & \chi_{\mathrm{ee}}^{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \overline{\overline{\chi}}_{\mathrm{mm}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \chi_{\mathrm{mm}}^{zz} \end{bmatrix}$$

$$\boldsymbol{J}_{\mathrm{s}} = \frac{\partial \boldsymbol{P}}{\partial t} + \nabla \times \boldsymbol{M}$$

$$J_y = (\epsilon_0 \chi_{\mathrm{ee}}^{yy} \frac{\partial E_y}{\partial t} + \epsilon_0 \chi_{\mathrm{ee}}^{yx} \frac{\partial E_x}{\partial t} - \frac{\partial(\chi_{\mathrm{mm}}^{zz} H_z)}{\partial x}) \hat{y}$$

$$E_y^n(i,j,p) = E_y^{n-1}(i,j,p) + \frac{\triangle t}{\epsilon_0} \left[\frac{Hx^{n-1/2}(i,j,p) - Hx^{n-1/2}(i,j,p-1)}{\triangle z} - \frac{Hz^{n-1/2}(i,j,p) - Hz^{n-1/2}(i-1,j,p)}{\triangle x} \right] - \frac{\triangle t}{\epsilon_0} \frac{J_y(i,j,p)}{\triangle z}.$$

$$E_y^n(i,j,p) = E_y^{n-1}(i,j,p) + \frac{\triangle t}{\epsilon_0} \left[\frac{Hx^{n-1/2}(i,j,p) - Hx^{n-1/2}(i,j,p-1)}{\triangle z} - \frac{Hz^{n-1/2}(i,j,p) - Hz^{n-1/2}(i-1,j,p)}{\triangle x} \right] - \frac{\chi_{\mathrm{ee}}^{yy}}{\triangle z} \frac{E_y^n(i,j,p) - E_y^{n-1}(i,j,p)}{\triangle t} - \frac{\chi_{\mathrm{ee}}^{yx}}{\triangle z} \frac{E_x^n(i,j,p) - E_x^{n-1}(i,j,p)}{\triangle t} - \frac{\triangle t}{\epsilon_0 \triangle z} \frac{\chi_{\mathrm{mm}}^{zz} H_z^{n-1/2}(i,j,p) - \chi_{\mathrm{mm}}^{zz} H_z^{n-1/2}(i-1,j,p)}{\triangle x}.$$



Because of the existence of the off-diagonal surface susceptibilities, the time differential equations of Ex and Ey are ***coupled*** for anisotropic case.

Matrix form of anisotropic SSM

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = \epsilon_0 \iint \frac{\partial \boldsymbol{E}}{\partial t} \cdot d\boldsymbol{s} + \iint \boldsymbol{J}_{\mathrm{s}} \cdot \delta(z) d\boldsymbol{s}$$

$$J_x = (\epsilon_0 \chi_{\mathrm{ee}}^{xx} \frac{\partial E_x}{\partial t} + \epsilon_0 \chi_{\mathrm{ee}}^{xy} \frac{\partial E_y}{\partial t} + \frac{\partial (\chi_{\mathrm{mm}}^{zz} H_z)}{\partial x}) \hat{y}$$

$$J_y = (\epsilon_0 \chi_{\mathrm{ee}}^{yy} \frac{\partial E_y}{\partial t} + \epsilon_0 \chi_{\mathrm{ee}}^{yx} \frac{\partial E_x}{\partial t} - \frac{\partial (\chi_{\mathrm{mm}}^{zz} H_z)}{\partial x}) \hat{y}$$

$$\begin{bmatrix} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \end{bmatrix} = \epsilon_0 \begin{bmatrix} 1 + \frac{\chi_{\mathrm{ee}}^{xx}}{\Delta z} & \frac{\chi_{\mathrm{ee}}^{xy}}{\Delta z} \\ \frac{\chi_{\mathrm{ee}}^{yx}}{\Delta z} & 1 + \frac{\chi_{\mathrm{ee}}^{yy}}{\Delta z} \end{bmatrix} \begin{bmatrix} \frac{\partial E_x}{\partial t} \\ \frac{\partial E_y}{\partial t} \end{bmatrix} + \frac{1}{\Delta z} \begin{bmatrix} \frac{\partial (\chi_{\mathrm{mm}}^{zz} H_z)}{\partial y} \\ \frac{\partial (-\chi_{\mathrm{mm}}^{zz} H_z)}{\partial x} \end{bmatrix}$$

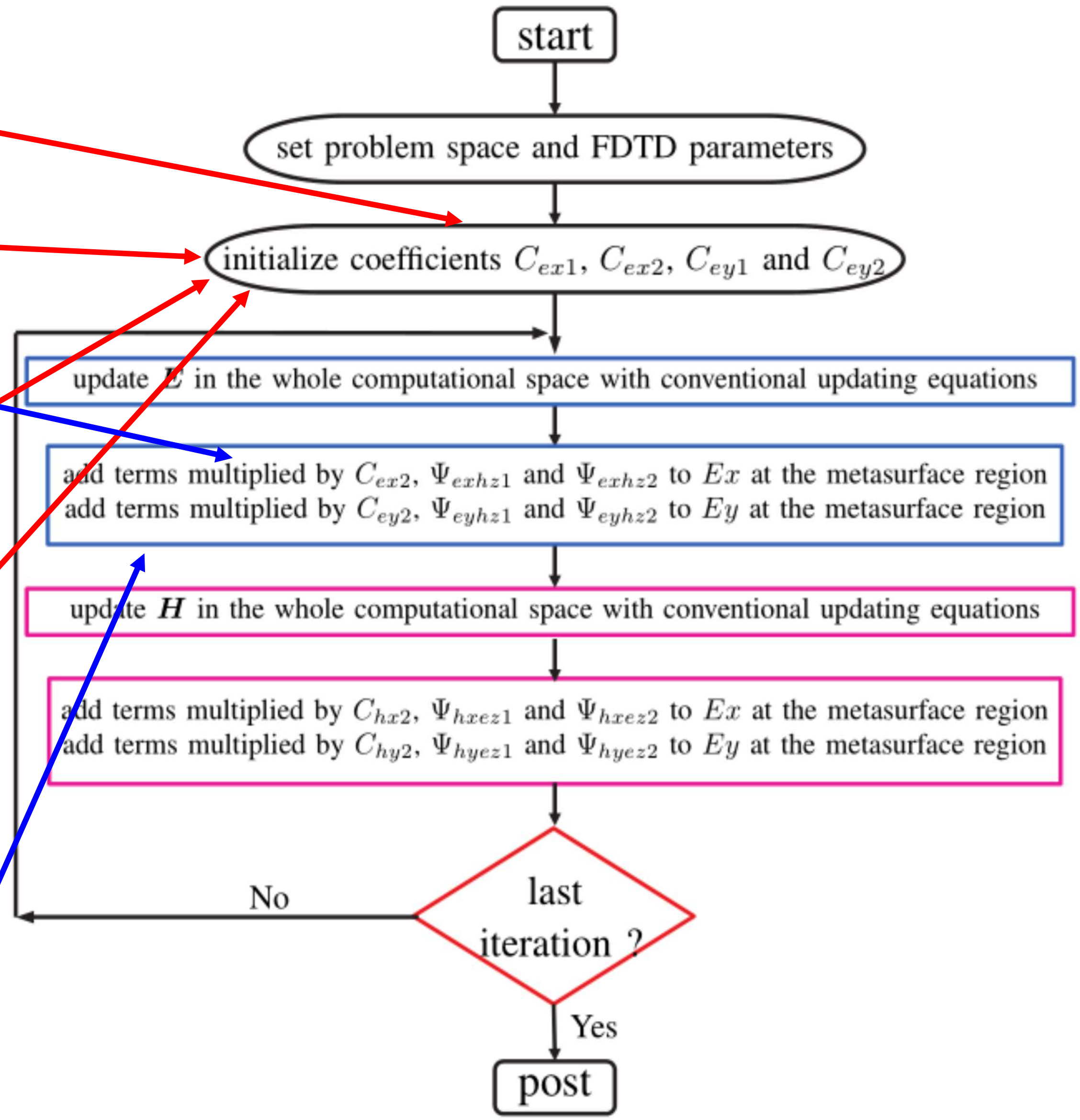
Matrix inversion

$$\begin{bmatrix} \frac{\partial E_x}{\partial t} \\ \frac{\partial E_y}{\partial t} \end{bmatrix} = \epsilon_0 \begin{bmatrix} 1 + \frac{\chi_{\mathrm{ee}}^{xx}}{\Delta z} & \frac{\chi_{\mathrm{ee}}^{xy}}{\Delta z} \\ \frac{\chi_{\mathrm{ee}}^{yx}}{\Delta z} & 1 + \frac{\chi_{\mathrm{ee}}^{yy}}{\Delta z} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \end{bmatrix} + \frac{1}{\Delta z} \begin{bmatrix} 1 + \frac{\chi_{\mathrm{ee}}^{xx}}{\Delta z} & \frac{\chi_{\mathrm{ee}}^{xy}}{\Delta z} \\ \frac{\chi_{\mathrm{ee}}^{yx}}{\Delta z} & 1 + \frac{\chi_{\mathrm{ee}}^{yy}}{\Delta z} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial (\chi_{\mathrm{mm}}^{zz} H_z)}{\partial y} \\ \frac{\partial (-\chi_{\mathrm{mm}}^{zz} H_z)}{\partial x} \end{bmatrix}$$

Flowchart of anisotropic FDTD-SSM

$$E_y^n(i,j,p) = E_y^{n-1}(i,j,p) + C_{ey1}\left[\frac{H_x^{n-1/2}(i,j,p) - H_x^{n-1/2}(i,j,p-1)}{\triangle z} - \frac{H_z^{n-1/2}(i,j,p) - H_z^{n-1/2}(i-1,j,p)}{\triangle x}\right] + C_{ey2}\left[\frac{H_y^{n-1/2}(i,j,p) - H_y^{n-1/2}(i,j,p-1)}{\triangle z} - \frac{H_z^{n-1/2}(i,j,p) - H_z^{n-1/2}(i,j-1,p)}{\triangle y}\right] + \Psi_{eyhz1} + \Psi_{eyhz2}$$

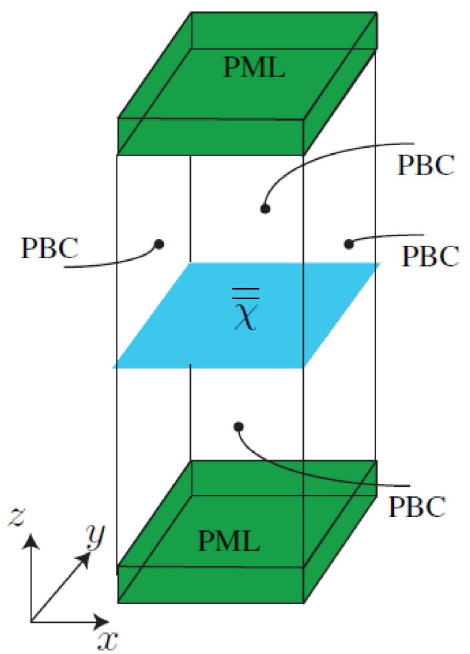
$$E_x^n(i,j,p) = E_x^{n-1}(i,j,p) + C_{ex1}\left[\frac{H_z^{n-1/2}(i,j,p) - H_z^{n-1/2}(i,j-1,p)}{\triangle y} - \frac{H_y^{n-1/2}(i,j,p) - H_y^{n-1/2}(i,j,p-1)}{\triangle z}\right] + C_{ex2}\left[\frac{H_z^{n-1/2}(i,j,p) - H_z^{n-1/2}(i-1,j,p)}{\triangle x} - \frac{H_x^{n-1/2}(i,j,p) - H_x^{n-1/2}(i,j,p-1)}{\triangle z}\right] + \Psi_{exhz1} + \Psi_{exhz2}$$

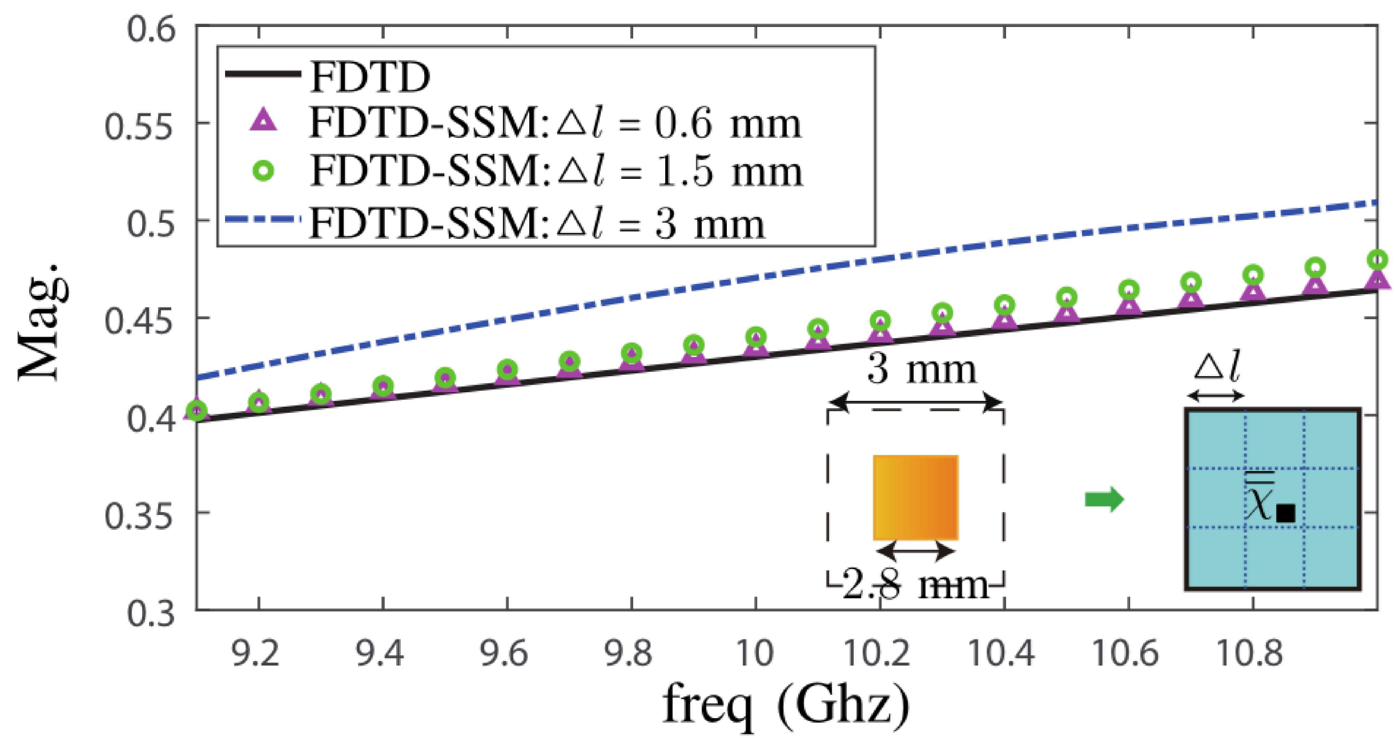


Outline

- Introduction of Metasurfaces
- Modeling of metasurface
- FDTD-SSM Algorithm
- Numerical Experiments
  - Uniaxial element
  - Biaxial element
  - General anisotropic element
- Conclusion

Uniaxial FDTD-SSM

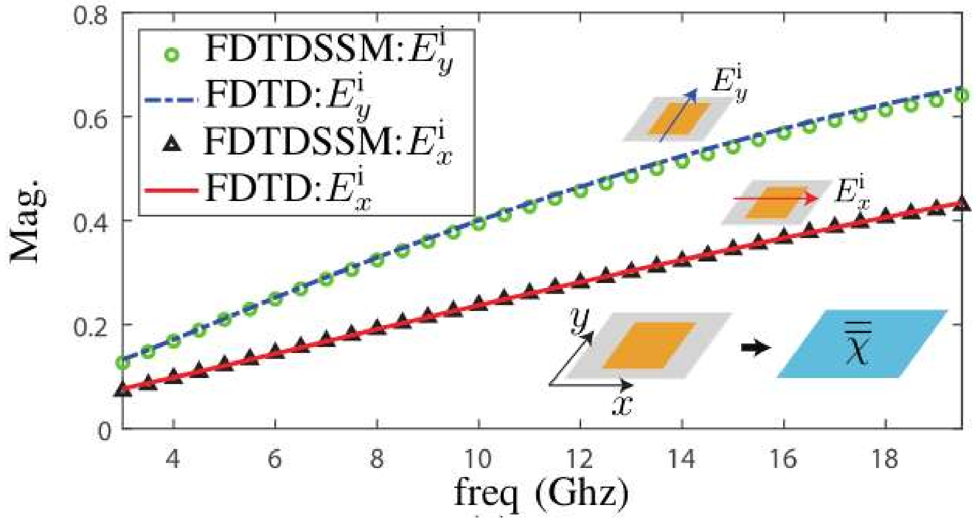




Remarks:

1. The meshing lattice size of conventional FDTD is $\frac{\lambda}{600}$ =0.05 mm.
2. As the meshing lattice size of FDTD-SSM decreasing to $\frac{\lambda}{50} = 0.6$ mm= 12×0.05 mm, its results overlap with that of conventional FDTD.

Biaxial FDTD-SSM



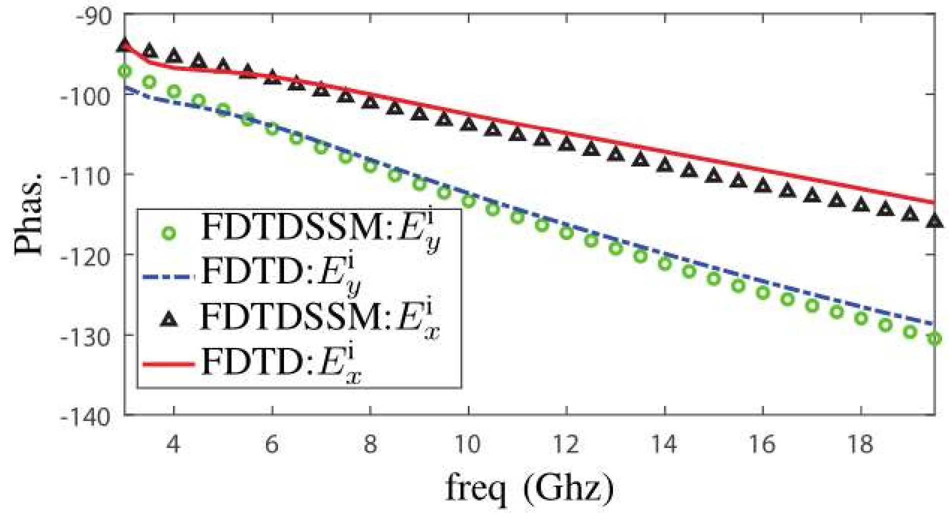


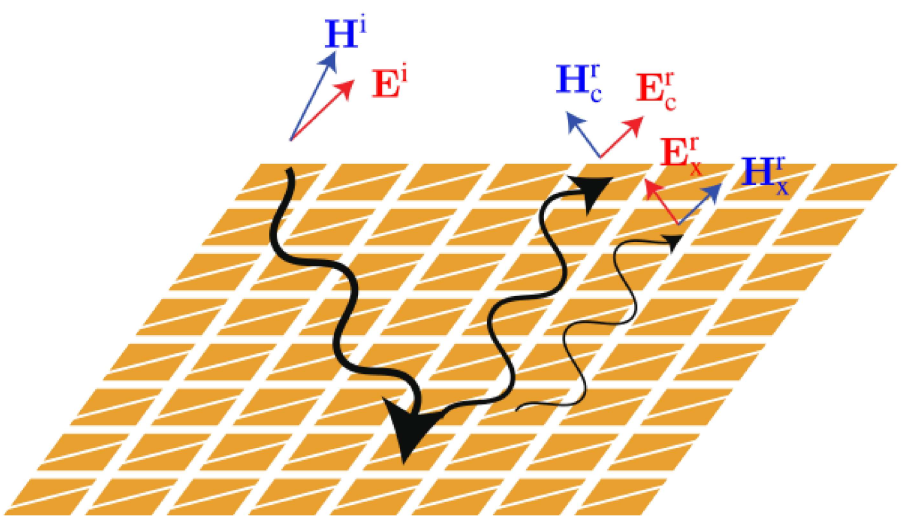
TABLE I
COMPUTATIONAL RESOURCES CONSUMPTION

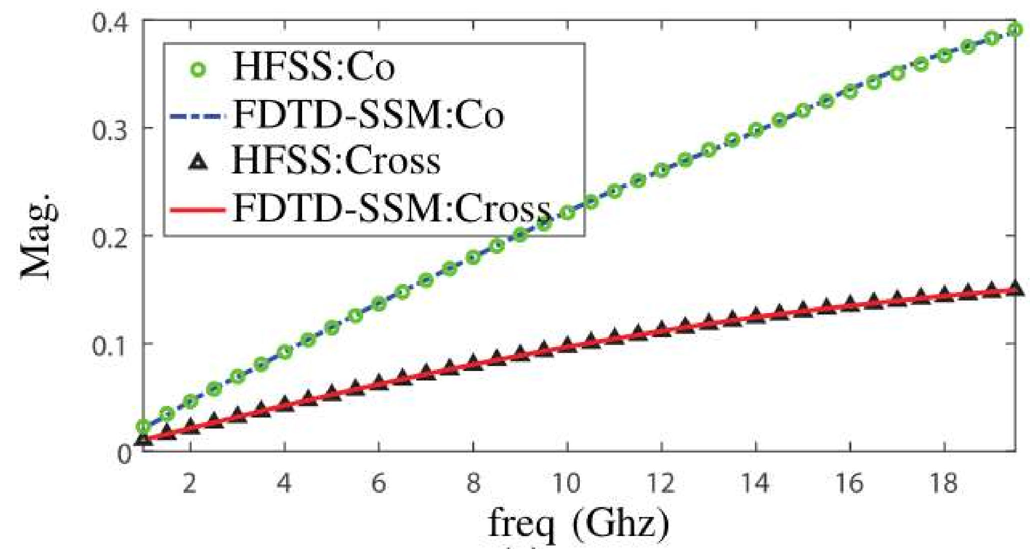
	cell size	time consum.
Conv. FDTD	$dx = dy = dz = 0.05$ mm	1410 s
FDTD-SSM	$dx = dy = dz = 0.75$ mm	3 s

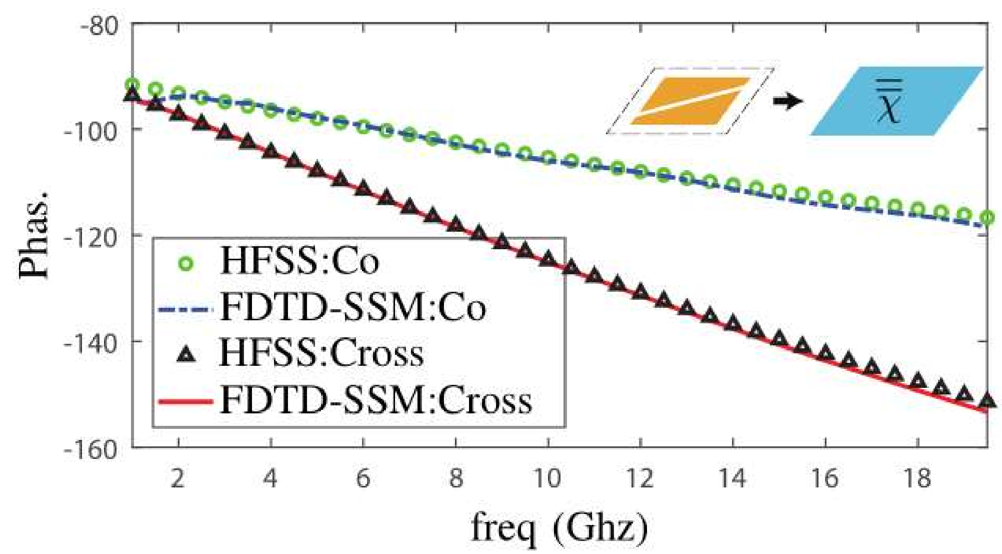
Remarks:

1. The FDTD-SSM algorithm is valid for different polarizations of incident waves.
2. The computational time is greatly saved by adopting FDTD-SSM.

Anisotropic FDTD-SSM







Remarks:

1. For anisotropic cases, cross polarization exists.
2. Both co- and cross- polarization are validated by HFSS.

Outline

- Introduction of Metasurfaces
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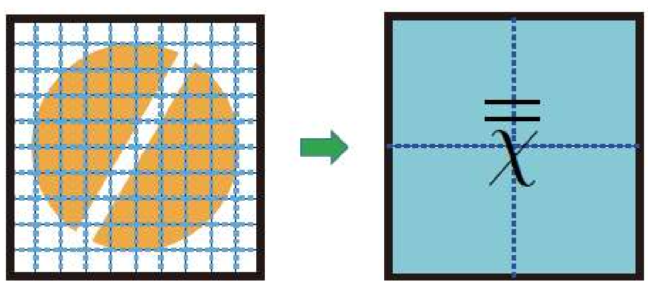
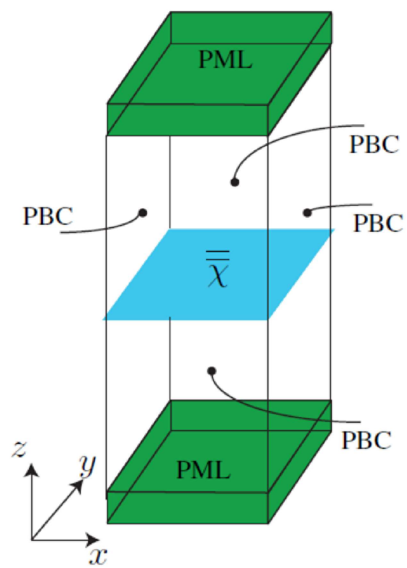
- Conclusion

Conclusion

- A novel FDTD-SSM algorithm for accelerating metasurface simulation.
  - Surface susceptibility model, Biaxial and anisotropic model, extraction
  - surface currents in FDTD
  - Advantages: simple & efficient
- Numerical experiments of the FDTD-SSM algorithm
  - Reduction of meshing lattice size, effectiveness for different shapes
  - Uniaxial, biaxial and anisotropic element shape
  - Consuming time is reduced from hours to seconds.





References

[1] C. L. Holloway, E. F. Kuester, J. A. Gordon, J. O'Hara, J. Booth, and D. R. Smith, "An overview of the theory and applications of metasurfaces: The two-dimensional equivalents of metamaterials," IEEE Trans. Antennas Propag., vol. 54, no. 2, pp. 10–35, 2012.

[2] X. Liu, F. Yang, M. Li, and S. Xu, "Analysis of reflectarray antenna elements under arbitrary incident angles and polarizations using generalized boundary conditions," IEEE Antennas and Wireless Propagation Letters,
vol. 17, no. 12, pp. 2208–2212, Dec 2018.

[3] X. Jia, X. Liu, F. Yang, M. Li, and S. Xu, "FDTD simulation of metasurfaces with generalized sheet transition conditions," in 2017 International Applied Computational Electromagnetics Society Symposium (ACES), Aug 2017, pp. 1–2.

[4] X. Jia, F. Yang, X. Liu, M. Li, and S. Xu, "Fast nonuniform metasurface analysis in FDTD using surface susceptibility model," IEEE Trans. Antennas Propag., 2018 (under review).

[5] A. Z. Elsherbeni and V. Demir, The Finite-Difference Time-Domain Method for Electromagnetics with MATLAB Simulations. The Institution of Engineering and Technology, 2016.

[6] F. Yang, J. Chen, R. Qiang, and A. Elsherbeni, "A simple and efficient FDTD/PBC algorithm for scattering analysis of periodic structures," Radio Sci., vol. 42, no. 4, pp. 3672–3672, 2007.

Thanks!